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ABSTRACT

The study examined whether embedding the linear equations and ratio and proportion strategies used to solve percent problems in the higher-order structures of proportional logic and group properties would aid achievement. Thus, it examined the interaction in differences in understanding fundamental structures in mathematics, and instruction requiring their use in solving percent problems. Participating were 141 eighth-grade students aged 12-14 from six intact classes. Three 10-item tests were constructed to assess achievement in solving percent problems; understanding of proportionality and group properties was clinically assessed with a balance beam task and a figure rotations task. Two sets of parallel units were prepared, one based on ratio and proportion concepts and the other on the solutions to linear equations. It was found that the attainment of formal operations for proportional logic was not associated with short-term success for solving percent problems with the ratio and proportion method, but it was for the delayed posttest 6 weeks later. For the linear equation method, both short- and long-term success required understanding of the structures for both group properties and proportional logic. Other results are also reported and discussed. (MNS)

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Path Analysis Model of Aptitude-Instruction Associations Over Time on Achievement in Solving Percent Problems

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Path Analysis Model of Aptitude-Instruction Associations Over Time on Achievement in Solving Percent Problems

A report of the National Assessment of Education Progress (NAEP, 1979) has cited declines in problem solving generally. Particular attention was called to problems dealing with the concept of percent.

> Only 35% of the 13 year olds and 58% of the 17 year olds knew what percent 30 is of 60, and only 8% at age 13 and 27% at age 17 could calculate 4% of 75 (NAEP, 1979, p.1).

Factors contributing to student difficulty, it was suggested, were deficiencies in instruction, failure of students to have the prerequisite knowledge, failure of students to grasp the underlying problem structure, and their inability to recognize systems in which the problems are embedded.

If the strategies used to solve percent problems are embedded in higher order structures and principles, then those individuals who have acquired them should better be able to call forth the particular procedures when needed. The current study sought to examine whether this was the case for the linear equations and ratio and proportion procedures used to teach the solution of percent problems. Both of these procedures were assumed to be embedded within the higher order structures of proportional logic and group properties. Another question raised in this study is whether an understanding of the properties of an Abelian group is associated with successful processing of these equations to solution.

Theoretical Framework

Relatively few empirical studies dealing with percent have been reported and those that have provided little insight. The late E. G. Begle in his book, Critical Variables in Mathematics Education stated:

> It is astonishing that there has been so little work in this area (percent). The findings are not sufficient to provide us with much guidance as yet, but they clearly indicate that further coordinated research along these general lines could have a substantial payoff (Begle, 1979, p. 61).

The research thus far has focused primarily on methods of instruction, of which four are common.

The conventional approach uses a rule for each of the three cases. The ineffectiveness of this approach is soon recognized by teachers when they provide students with problems that unite each category.

A second approach utilizes a formula: base × rate = percentage. Students are expected to substitute the values for two of the variables then find the third. According to Johnson and Rising (1967), a deficiency of this procedure is that students have difficulty distinguishing between base and percentage.

A third more popular approach is based upon the view of a percent as a ratio whose base for comparison is 100. The missing number is found through the use of a proportion. Methods such as finding equivalent ratios and cross-products are introduced. Proponents of this approach (Johnson and Rising, 1967; Nelson, 1969; Brown and Kinney, 1973; Cole and Weissenfluh, 1974; Schmalz, 1977) prefer it because they claim it is based on the meaning of percent and a proportion is used for all three cases.

A fourth strategy, the linear equation method, presents percent as another name for a decimal or fraction. The method of solution utilizes an open sentence in the form $A \cdot \underline{X} = B$ for all three cases. Advocates of this approach (Nyberg, 1947; Volperl, 1954; Kessler, 1960) contend that the method is easier than the three case method. Students can solve all three cases using a single equation; further, it is likely that they have had elementary school experience with equations.

Three studies that focused on comparisons that included these instructional methods have been reported in the literature.

In one study (McMahon, 1960), consisting of 245 seventh graders, the conventional and ratio and proportion methods were compared. Differences, when found, favored the ratio and proportion method.

A second study (May, 1965) with 156 seventh graders compared the effectiveness of three procedures: conventional, ratio and proportion, and discovery. For immediate learning discovery fared best with ratio and proportion following. For retention, ratio and proportion ranked first.

In a third study (Bidwell, 1969) with 566 fifth graders with mean IQ 125, some were taught by the ratio and proportion method and others by the linear equation. Another variable in the study was mode of teacher presentation, direct or indirect. The study found no significant interactions or main effects.

All of the cited studies employed different procedures for the solution of percent problems. In each, students were taught to use certain methods that resulted in the different representations of the problem. The use of particular processes of solution raises an interesting question. What is the step between problem statement and problem representation? Gerald Kulm suggests:

> Heuristic processes form the bridge between the problem statement and a problem representation as well as affecting the transition to related problem statements and representations (Kulm, 1979, p. 19).

If problem representation is a key element, it seems reasonable to expect subjects who have acquired more elaborate representations for a particular structure to be more successful.

Chi, Glaser and Rees report that knowledge of the deep structure

of the problem serves to distinguish expert from novice problem solvers. Referring to a series of problem-solving studies in physics, they report:

> The categorization studies show that without actually solving the problems, and in less than 45 seconds, experts were able to encode the problem into a deep level of representation, one that enables them to grossly determine the solution method applicable to the problem (Chi et al., 1981, p. 65).

These researchers also define different levels of structure in the knowledge of an individual. Experts, they indicate, organize their knowledge around principles while novices do so around objects, that is,

> Experts possess schemas of principles that may subsume schemas of objects, whereas novices may possess only schemas of objects (p. 85).

It is a hypothesis of this study that the strategies used to solve percent problems are embedded in the higher order structures of proportional logic and group properties. The rationale for identifying these as higher order comes from the work of Inhelder and Piaget (1958) and Bruner (1961).

Careful descriptions of the development of proportional logic schemes in children have been provided by Inhelder and Piaget in The Growth of Logical Thinking. In an experiment using a balance beam, they studied how the proportionality scheme is linked with the equilibrium scheme. They report:

> As a result of previous research we know that in all realms (space, speed, chance, etc.) the notion of proportions does not appear until formal substage III-A (Inhelder and Piaget, 1958, p. 164).

They distinguish between mathematical proportions, the equality of two ratios, and logical proportions which derive from group (INCR)

and lattice structures. The fact that 8 to 11 year old subjects have been unable to discover the equality of two ratios which form a proportion raises some question. They write:

> It is not enough to explain it as a result of timing in the academic curriculum. In the first place, we have seen subjects construct the notion in the experimental situation before they have learned it in school. Secondly, if it could be understood earlier, we can be sure that its presentation in the academic program would accelerate the timing of acquisition! Thus we have to look for the explanation of its late comprehension in the actual structure of the operations available to the subjects at the different levels (p. 314).

The examination of the structure of the operations used to solve the balance scale problem led Inhelder and Piaget to conclude that the construction of the proportions greater weight at shorter distance from fulcrum balanced by smaller weight at longer distance was linked to those schemes that are based on the INRC group. They report:

> Where does the generalization found at substages III-A and III-B (formal) come from? Without doubt this is where the notions of compensation and reciprocity connected with the INRC group come in (p. 177).

One would expect, then, that those individuals who provided evidence of formal operations on the balance scale problem would be able to construct mathematical representations for problems whose structures are embodied in these schemes. With this view, performance on that task was examined in relation to a student's ability to construct mathematical representations associated with either the linear equations or ratio and proportion strategies and translate these into an appropriate algebraic equation. Also, the resulting equations yielded by either methods can be shown, using Goldin's criteria (1979), to be structurally equivalent.

Given that an individual is able to construct a mathematical

representation of a problem, it is of interest to study what schemes are related to the successful processing of that statement to a solution.

Bruner contends that understanding of commutativity and associativity is essential for the successful solution of a linear equation. In a similar vein, the identity and inverse properties can be shown to be inherent to understanding the process of solution.

Objective

The study has as its objective an examination of the interaction of differences in understanding of fundamental structures in mathematics and instructional treatments requiring their use to the solving of percent problems. The independent variables in the study were achievement on two tasks and the dependent variable was correctly solving percent problems. For the six-week evaluation, two measures of the dependent variable were used: in one case, correctly setting up the problem and in another, correctly solving it.

Specifically, the study sought to examine the relationship between (a) those variables judged to be associated with the structures of the concepts underlying both treatments: understanding of proportionality and the properties of an Abelian group; (b) methods of instruction: ratio and proportion and linear-equations; and (c) achievement in solving percent problems, immediately and six-weeks after instruction.

Data Source

The data were collected in conjunction with a series of coordinated research studies of the Rutgers University-Middlesex Public Schools Math Project (Maher, C. A., O'Brien, J. J., Heller, L., Smith, J. Normandia, B.,

& DeStefano, F., 1982). The population consisted of 141 eight grade students whose ages ranged from 12 to 14 years and whose I.Q.'s ranged from 83 to 128. Six intact classes and two teachers participated in the study. At random, each teacher was assigned three classes that were designated as linear-equation, ratio and proportion, and control (no percent instruction). Except for one advanced class who studied Algebra, the school did not group students in mathematics according to ability. Analyses of (1) achievement test scores, (2) prior success in mathematics, and (3) I.Q. scores supported the assumption of heterogeneity within classes and homogeneity among groups.

Methods and Techniques

Since no instruments for measuring achievement in solving only percent problems were available, three tests - one for pretesting, and two others for post and delayed posttesting six weeks later - were constructed and pilot tested. The items were evaluated for content validity by five mathematics teachers who had taught percents. All three tests consisted of ten problems involving the three cases of percent. Reliability coefficients of 0.93, 0.90, and 0.89, respectively, were computed using the Kuder-Richardson Formula 21 index. Subjects were instructed to solve each problem and show all work. Five mathematics instructors, graduate students in mathematics-education at Rutgers, scored the test for (1) correctly stating the algebraic equation, (2) processing it and (3) correctly solving the problem. For the immediate posttest, a minimum requirement for success was that seven out of 10 problems, including two-thirds of each case, were correct. For the six-week posttest, that same criterion was applied for (1) correctly setting up the problem and (2) correctly solving it.

Understanding of proportionality and group properties structures were assessed clinically for each student prior to instruction. For proportionality, a balance beam task, based upon Inhelder and Piaget's equilibrium of the balance (1958), was used; for the group properties, a figure rotations task was constructed. Protocols for both tasks were developed and field tested prior to the study over a range of three grades - seven to nine. Rigorous training and evaluations of clinicians were provided to maintain high standards of consistent performance. Using a simultaneous scoring of video-taped interviews for these tasks, a 0.95 measure of inter-clinician reliability was obtained.

Balance Beam Task

Successful performance on the balance beam task (Inhelder and Piaget, 1958) required that the subject balance a beam by placing unequal weights at unequal distances from the fulcrum. A carefully selected series of problems was posed by a clinician who allowed subjects to construct the relationship of proportionality between weight and distance if it were not spontaneously forthcoming.

Consistent with the criterion of Inhelder and Piaget, subjects who were classified in the formal substages III-A and III-B were regarded as having acquired the proportional schemes. Hence a successful or high rating was assigned if there was evidence of formal operations for proportionality; otherwise the rating was given as low.

Group Properties Task

The absence or presence of understanding of the structure of the properties of an Abelian group, i.e. an embodiment of closure, associative, identity, inverse and commutative properties, were assessed by

evaluating a subject's performance on a task requiring the rotation of two figures which was developed at Rutgers (Maher, et al., 1982). In order that the assessed understanding be of the structure of these properties, it was decided that possibly deceptive clues such as conventional numbers and symbols not be used.

The task became a two-stage operation, the first devoted to teaching and the second to assessment, both being individually administered to subjects by a trained clinician. The materials employed were two small dolls, boy and girl, capable of being turned zero and 180 degrees. Four instructions: "only boy turns", "only girl turns", "both turn" and "nobody turns" were placed on "command cards" and placed along the top and side of a 4 by 4 matrix. The members of the set were the four "turns" and the operation was "followed by." Using a protocol and record sheet, it was the clinician's objective in the first stage to assist (without giving answers) the student to find the correct answers and complete the matrix. When it was completed, the second stage, assessment, began. Here the training of clinicians became especially important for success with the task required the student to describe the structure of the five properties of an Abelian group.

Two orders of response were considered successful: (1) immediate generalization of the property and ability to give an appropriate specific example and (2) no immediate generalization but the framing of a generalization and provision of examples after the clinician directed the subject's attention to a specific example. All other responses were considered unsuccessful. The subject was rated as successful or high on the task if either all five properties were identified at the first level of questioning or four were identified at the first level and the fifth at the second level, that is, after a single probe. Otherwise a low rating was assigned.

The Curriculum

All four classes began the study, eight weeks in duration, the day after the pretest was administered.

Two sets of parallel units for teaching percent, one based on ratio and proportion concepts and another based on the solutions to linear equations, were developed. The first eight lessons, dealing with prerequisite material identical for both treatment groups, focused on the meaning of a variable, inverse operations for addition and multiplication, open sentences, and the use of flow charts for writing algebraic expressions.

Units 9-14 were specifically prepared for each group. The linear-equation lessons used flow charts to solve open sentences of the form $\underline{X} + A = B$ and $A \cdot \underline{X} = B$. The ratio-proportion lessons dealt with the meaning of a ratio, equivalent ratios, and the meaning of proportion. The principle that in a proportion the product of the means is equivalent to the product of the extremes was developed to direct students to find an unknown term in a proportion.

Units 15-21 were identical for both treatments. They focused on the meaning of percent and the equivalent representations of percents using fractions and decimals.

The final three units, 22-24, were specifically prepared for each group. They were designed to introduce the structure of each of the methods, linear-equation and ratio-proportion. Modified versions of the 24 units can be found in Math 1 (Maher, C., Rosenauer, W., Maher, J., Shay, K., and Snyder, A., 1981) except for units 22-24 for the ratio and proportion method, developed by DeStefano (1981).

For both methods, the mode of instruction was teacher-directed, inductive, and symbolic. Concrete activities were provided where possible. The goal of instruction was to provide students in both treatments with

presentations that differed only in the representation that was to be used for solving the problem. It was the intent that students be given activities that would lead them to explore, experiment, and generalize to the particular model that was the object of that instructional treatment.

Results

The data were analyzed using loglinear and conditional odds procedures and described graphically using a path analysis. The discrete nature of the data and the multivariate relationships suggested this model as the most appropriate.

Table 1 compares the success rate of the experimental and control groups on the pretest, posttest, and delayed (six-week) posttest. Contingency table analyses based upon the Chi-Square statistic using the data from Table 1 support the claim that the treatment and control groups had similar but low (0 to 4.3%) success rates on the percent pretest.

Insert Table 1 about here

For both the treatment groups, more than half of the students were classified as successful on the immediate posttest. The ratio and proportion treatment had a significantly greater percentage of students (11.3) who were successful on the percent test than for the linear equation treatment group (64.8% vs. 53.5%). On the six-week delayed posttest, the success rate was approximately one-half of what it was on the posttest. Additionally, no statistically significant difference in the success rate of the two groups was observed on the six-week delayed posttest ($\chi^2 = 1.29$, $df = 1$).

Loglinear analyses using the Biomedical Computer Program (BMDP, 1979) were used to model the relationship between percent achievement and student performance on the group properties and balance beam tasks. Table 2 summarizes the results indicating that regardless of the teaching method, students classified as high on both tasks had the highest odds of percent achievement followed by those classified as high on only one task.

Insert Table 2 about here

The group classified as low on both tasks had the lowest odds of success.

Table 3 lists the conditional relative odds of percent achievement for high vs. low group properties and proportional reasoning groups.

Insert Table 3 about here

The top of Table 3 shows that students classified as high in understanding the group properties had 4.5 times the odds of percent achievement on the posttest and 5.3 times on the delayed posttest as those with like proportional reasoning ratings but a low group properties rating. The bottom half of Table 3 shows that students who were classified as high in proportional reasoning had 7.9 times the odds of percent achievement on the posttest if taught by the linear equation method, 1.2 times the odds of achievement if taught by the ratio and proportion method, and 4.8 on the delayed posttest as students with like group properties ratings but low proportional reasoning rating.

Figure 1 summarizes the analyses of Table 3 of the conditional relative odds of percent achievement for the high vs. low group properties and proportional reasoning assessment groups.

Insert Figure 1 about here

The upper half of Figure 1 in conjunction with the data from Table 3 provide an explanation of why the ratio and proportion method resulted in a higher rate of success on the posttest than the linear equation method. It can be observed that the ratio and proportion method is not strongly related to an understanding of proportional reasoning for short term success as measured on the posttest (note a path value of 1.2) while the linear equation method is highly related to an understanding of proportional reasoning for short term success (note a path value of 7.9). Both methods of instruction, however, are associated with group properties understanding (note a path value of 4.5).

With regard to the six-weeks delayed posttest achievement, success is statistically independent of the method of instruction but dependent on group properties understanding (note a path value of 5.3) and proportional reasoning (note a path value of 4.8). For example, a student who is classified as high on both group properties understanding and proportional reasoning had 5.3 × 4.8 or 25.44 times the odds of achievement as a student who was assessed as low on both tasks.

In order better to understand the significance of the associations of the group properties and proportional reasoning tasks, problem success was partitioned into two parts: (1) successful set up, that is, correct translation of the statement of the problem into an algebraic linear equation and (2) successful processing of the equation to correct solution.

Table 4 gives the relationship of success on problem set up and correct solution (following successful set up) conditioned on the subject's proportional reasoning and group properties assessments.

Insert Table 4 about here

Table 4 shows that the probability of successful problem set up is related to both the subject's proportional logic and group properties assessments.

Students assessed as formal on the proportional logic task had at least twice the chance of successful problem set up as those rated as nonformal. Those successful on both tasks were found to have a higher chance of correct set up as those successful only on the proportional logic task. Subjects with a nonformal proportional logic assessment, regardless of their group properties rating, had the lowest problem set up success rate.

Achievement of a correct answer following successful set up is statistically dependent on the stubject's group properties assessment but independent of the subject's proportional reasoning level. Students assessed as high on the group properties task had approximately twice the chance of obtaining the correct answer after a successful set up than those who were low.

Discussion

While the attainment of formal operations for proportional logic was not associated with short term success for solving percent problems with the ratio and proportion method, it was for the delayed posttest six-weeks later. That is, students who retained the method were those with formal operational schemes for proportional logic. It may be that students who were immediately successful but not at six-weeks may have learned a procedure only partially understood or perhaps understood only to the extent the development of the proportionality schemes allowed. This result may help explain the popularity of the ratio and proportion method. Teachers, focusing only on short-term success are perhaps erroneously inferring student

understanding of the method of solution.

For the linear equation method of instruction, however, both short and long-term success required understanding of the structures for both the group properties and proportional logic. One interpretation of this result is that students cannot as easily be "trained" to apply this method.

In the long run, it may be that method of instruction becomes irrelevant whereas attainment of understanding of the structures in which the procedures are embedded becomes primary.

For example, the significance of understanding the structure of the properties of a group for processing the equations yielded by either method of instruction is not surprising. Solutions to both equations are based upon knowledge of these properties.

The significance of the strong association of success on both tasks to successful problem set up is of interest. One would expect subjects who have acquired formal operation schemes for proportional logic to be able to come up with the appropriate equation for the proportion method. Perhaps the attainment of the INRC group logic manifested in the accomplishment of the balance beam task can serve as explanation for the successful statement of the linear equation.

A less apparent relationship is that of the group proerties task to successful problem set up. Perhaps Goldin's (1982) proposal of the "non-verbal symbolization" step in processing a problem may provide some insight. He has suggested that this step, often overlooked, may be preliminary to problem representation. The highly spatial nature of the group properties task may require that successful individuals create such a symbolization of the problem in order to reach a solution. Future research might be directed to an analysis of this task particularly as it relates

to understanding the "non verbal symbolization" process that may be taking place. A separation of the component of the task that deals explicitly with the structure of the group properties would also be helpful.

Finally, it is suggested that future efforts be directed to more detailed analyses of the processes involved in developing the structures associated with methods of solutions to problems rather than focusing on the methods alone.

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TABLE 1

PERCENT ACHIEVEMENT VS. INSTRUCTIONAL METHOD

INSTRUCTIONAL METHOD	SAMPLE SIZE	PERCENT CLASSIFIED AS SUCCESSFUL		
		Pretest	Posttest	Delayed Posttest
Ratio & Proportion	42	0.0	64.8	38.6
Linear Equation	46	4.3	53.5	27.3
Control	53	3.8	3.8	3.8

TABLE 2

LOGLINEAR MODEL ESTIMATES OF THE ODDS OF SUCCESS AS A FUNCTION OF THE STUDENTS GROUP PROPERTIES AND PROPORTIONAL REASONING ASSESSMENTS

Group Properties	Proportional Reasoning	ODDS OF SUCCESS			
		Posttest		Delayed Posttest	
		LE	RP	LE	RP
H	H	12.01	8.18	2.59	2.59
L	H	2.67	1.82	0.49	0.49
H	L	1.53	6.86	0.54	0.54
L	L	0.34	1.53	0.10	0.10

TABLE 3

COMPARISONS OF THE ODDS OF PERCENT ACHIEVEMENT FOR STUDENTS WITH HIGH VS. LOW SCORES FOR THE GROUP PROPERTIES AND PROPORTIONAL REASONING TASKS

High vs. Low Group Properties Relative Odds[a] of Percent Achievement Conditioned on Student's Proportional Reasoning Score

Proportional Reasoning	Relative Success Odds		
	Posttest		Delayed Posttest
	LE	RP	LE/RP
H	4.5	4.5	5.3
L	4.5	4.5	5.3

High vs. Low Proportional Reasoning Relative Odds[b] of Percent Achievement Conditioned on Student's Group Properties Score

Group Properties	Posttest		Delayed Posttest
	LE	RP	LE/RP
H	7.9	1.2	4.8
L	7.9	1.2	4.8

[a]The ratio of success odds from Table 2 of the high vs. low group properties scores with like proportional reasoning scores

[b]The ratio of success odds from Table 2 of the high vs. low proportional reasoning scores with like group properties scores

TABLE 4

DELAYED POSTTEST SCORES PARTITIONED ACCORDING TO SUCCESSFUL PROBLEM SET UP AND PROCESSING

Proportional Reasoning	Group Properties	Sample Size	Probability of Successful Set Up	Probability of Correct Answer Following Set UP
L	L	32	0.31	0.40
L	H	28	0.39	0.80
H	L	10	0.70	0.33
H	H	19	1.00	0.83

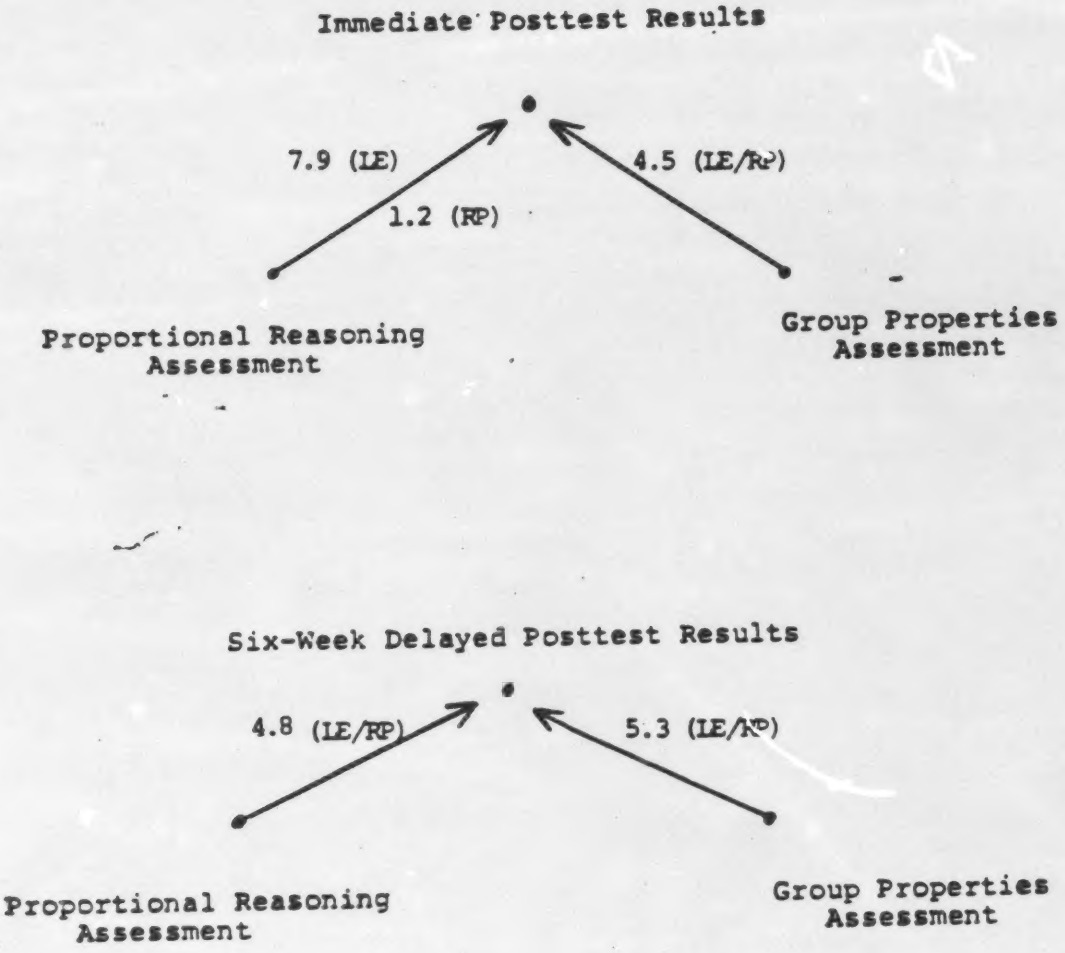

Figure 1. Path analyses showing the relationship between percent achievement and proportional reasoning and group properties assessments. (A path value, say r, on the vector connecting two assessments, say A and B, means that a subject who is high on A has r times the odds of being high on B as a subject who is low on A)